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Zero-bias current in a single-electron transistor with identical trapezoidal tunnelling barriers for finite fixed gate voltages

H Nakashima and K Uozumi†

Department of Physics, College of Science and Technology, Aoyama Gakuin University, 6-16-1 Chitosedai, Setagaya, Tokyo 157-8572, Japan

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Abstract. We predict that a single-electron transistor (SET) whose two tunnel junctions are identical asymmetric tunnelling barriers (ATBs) can give rise to a finite tunnelling current, at finite temperatures and for finite fixed gate voltages, in the absence of an applied voltage. This new theoretical transport property of a SET with identical ATBs is derived from the orthodox discussion of the Coulomb blockade oscillations, taking account of asymmetry in the potential shape of the tunnelling barriers. In this paper we use trapezoidal tunnelling barriers as the ATBs to make our proposed SET, and simulations of the SET are performed on the basis of the semiclassical approach.

It is well known that an asymmetric tunnelling barrier (ATB), whose potential profile is asymmetrical, gives a voltage offset in the conductance minimum [1, 2]; that is, a larger tunnelling current flows from one electrode of an ATB to the other when the electrode of lower internal workfunction is positively biased, which we call an asymmetric current–voltage characteristic. The asymmetry in the current–voltage characteristic arises solely from asymmetry in the potential barrier shape. It has previously been theoretically [3] and experimentally [4] demonstrated that two low-capacitance tunnelling junctions in series can produce a large degree of asymmetry in the current–voltage characteristic outside the Coulomb blockade region when at least one of the two junctions is an ATB instead of a symmetric tunnelling barrier (STB). The two-junction system produced in the experiment described in [4] consisted of a STB and an ATB, and a GaAs–AlGaAs heterostructure was used as its ATB. In our previous paper [5], quantitative simulations of a single-electron transistor (SET) with one ATB were first presented. The present paper predicts another interesting feature of a SET with two identical ATBs: that, in the absence of an applied voltage, a finite tunnelling current flows at finite temperatures and for finite fixed gate voltages.

We here take identical trapezoidal potential (tunnelling) barriers as ATBs, and these are set in the same direction to make a SET as shown in figure 1. We assume that there is no zero-bias misalignment of the Fermi levels in this set-up. The semiclassical tunnelling model is used to describe the single-electron transport, and simulations of our SET are

† Author to whom any correspondence should be addressed; telephone: (03) 5384 1111, ext 3104; fax: (03) 5384 6100; e-mail: uozumi@phys.aoyama.ac.jp.

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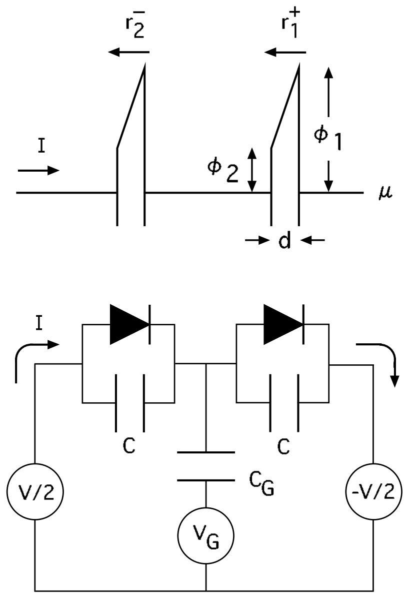

Figure 1. An energy diagram of trapezoidal potential barriers (above) and a schematic diagram of a SET with identical ATBs (below).

carried out in terms of the analytic solution described in [6]. Using Kirchhoff's law for two loops in the circuit, the equations for the voltage V_j across the jth tunnel junction are given as

$$\begin{aligned} V_1 &= \frac{1}{2}V + \frac{C_G}{C_\Sigma}V_G - \frac{Ne}{C_\Sigma} \\ V_2 &= \frac{1}{2}V - \frac{C_G}{C_\Sigma}V_G + \frac{Ne}{C_\Sigma} \end{aligned} \tag{1}$$

where V is the bias voltage, V_G is the gate voltage, C_G is the gate capacitance, N is the number of extra electrons in the inner electrode, and $C_\Sigma = 2C + C_G$. Also, C is the tunnel junction capacitance. Assuming that the transmission coefficient $T(E_z)$ for an electron of energy E_z in the z-direction perpendicular to the barriers is much less than unity for given V, V_G, and N, the rate of electron tunnelling $r_j^\pm$ through the jth junction, where the index $\pm$ relates to the change $N \to N \pm 1$, is represented by a golden rule equation [7]:

$$r_j^\pm(N) \propto nev_z = \frac{2e}{(2\pi)^3}\int \mathrm{d}^2\boldsymbol{k}_t\ \mathrm{d}k_z\ \frac{1}{\hbar}\left(\frac{\partial E_z}{\partial k_z}\right)T(E_z)f(E-\mu)[1-f(E-\mu-\Delta E_j^\pm)] \tag{2}$$

where v_z is the group velocity in the z-direction, $\boldsymbol{k}_t$ is the transverse momentum parallel to the x–y plane, $f(E)$ is the Fermi distribution function, μ is the chemical potential of the electrodes, and $-\Delta E_j^\pm$ is the free-energy change of the system associated with electron tunnelling across the jth junction. The equations for $\Delta E_j^\pm$ are

$$\begin{aligned} \Delta E_1^\pm &= \frac{e}{C_\Sigma}\left[\frac{e}{2} \pm (Ne + C_G V_G) \mp \left(C + \frac{C_G}{2}\right)V\right] \\ \Delta E_2^\pm &= \frac{e}{C_\Sigma}\left[\frac{e}{2} \pm (Ne + C_G V_G) \pm \left(C + \frac{C_G}{2}\right)V\right]. \end{aligned} \tag{3}$$

Equation (2) can be rewritten as

$$r_j^{\pm}(N) = \frac{2e}{(2\pi)^3}\frac{1}{\hbar}\int_0^{\infty} \mathrm{d}E_z\ T(E_z)\int_{-\infty}^{\infty} \mathrm{d}^2\boldsymbol{k}_t\ f(E-\mu)[1 - f(E-\mu-\Delta E_j^{\pm})]. \tag{4}$$

According to the relations $E_z = \hbar^2 k_z^2/2m^*$ (where m^* is the effective mass), and $E = \hbar^2(k_x^2 + k_y^2 + k_z^2)/2m^*$, equation (4) is numerically solved for given V, V_G, and N. We precisely calculate $T(E_z)$ by solving the discretized one-dimensional Schrödinger equation

$$\frac{\hbar^2}{2m^*}\left[\frac{\Psi(z+s) - 2\Psi(z) + \Psi(z-s)}{s^2}\right] = [U_j(z, V_j) - E_z]\Psi(z) \tag{5}$$

where s is the step size and $U_j(z, V_j)$ is the potential profile of the jth tunnelling barrier in the z-direction, where z is measured from the first interface of the jth tunnelling barrier, crossed by a tunnelling electron, to the second interface. Using the algorithm described in [8], the wavefunction at the position $z = 0$ on the jth tunnelling barrier can be solved as

$$\Psi(0) = Au(+) + Bu(-)$$

where $u(+)$ and $u(-)$ represent the incident and reflected wave, respectively. The transmission coefficient is then given by

$$T(E_z) = 1 - |B|^2/|A|^2. \tag{6}$$

Here we write the potential profiles U_j as

$$U_j(z, V_j) = \begin{cases} \mu + \phi_1 + (z/d)(\phi_2 - eV_j - \phi_1) - \Delta\phi(z) & (\text{for } r_1^+ \text{ and } r_2^-) \\ \mu + \phi_2 - (z/d)(\phi_2 - eV_j - \phi_1) - \Delta\phi(z) & (\text{for } r_1^- \text{ and } r_2^+) \end{cases} \tag{7}$$

where ϕ_1 and ϕ_2 are the high and low internal workfunctions, respectively, d is the barrier thickness, and $\Delta\phi(z)$ is the image potential in the barriers. Here $\Delta\phi$ is approximately given as [9]

$$\Delta\phi(z) = 5.75d/[\kappa z(d-z)]\ (\mathrm{eV}) \tag{8}$$

where the dimensions are in ångströms and κ is the high-frequency dielectric constant.

We are here interested in an anomalous current versus gate voltage (I–V_G) characteristic of our proposed SET in the absence of an applied voltage, and therefore give a simple argument regarding the new physical effect. Let us now set the bias voltage V to zero. We shall restrict ourselves to the range $[0,\ e/C_\Sigma]$ of offset voltages applied at the inner electrode since $m(e/C_\Sigma)$ with $m = 0, \pm 1, \pm 2, \ldots$ are equivalent offsets. When the Coulomb voltage $V_c = e/2C_\Sigma$ is applied across the second tunnel junction by means of the gate voltage ($V_G = -e/2C_G$) at nonvanishing but low temperatures, $T \ll \Delta E_j^{\pm}$, well defined Coulomb blockade oscillations in the energy levels of the inner electrode occur. (If $T = 0$, no electron tunnelling events occur.) Then the possible charge states of the inner electrode are only $N = 0$ and 1, which correspond to the Fermi energies, $-eV_c$ and $+eV_c$, of the inner electrode, respectively, measured relative to the Fermi energy of the outer electrodes. The ensemble distribution $\sigma(N)$ as a function of N can be obtained by the requirement for a transition between the two adjacent charge states in a steady state:

$$\sigma(0)[r_1^+(0) + r_2^+(0)] = \sigma(1)[r_1^-(1) + r_2^-(1)] \tag{9}$$

where

$$\sum_{N=-\infty}^{\infty} \sigma(N) = 1.$$

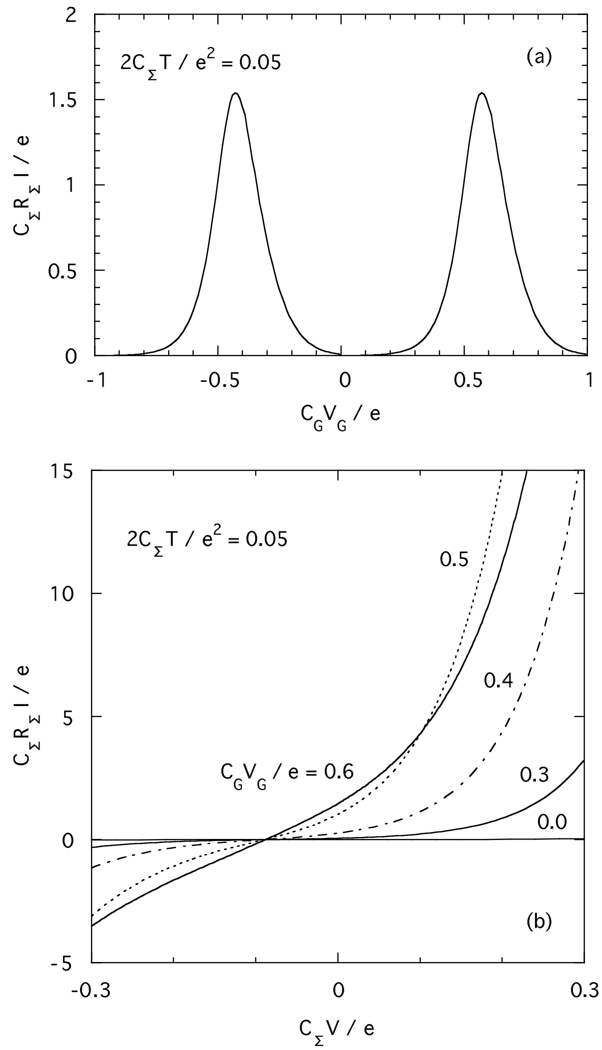

Figure 2. Simulated transport properties of the SET with identical trapezoidal tunnelling barriers. $R_\Sigma = 2R$, where R is the tunnelling resistance per junction for very low bias voltages and temperatures. (a) I–V_G characteristics for $V = 0$. (b) I–V characteristics for different fixed values of V_G.

Because $V = 0$, we have $r_1^+(0) = r_2^-(1)$ and $r_2^+(0) = r_1^-(1)$. Substituting these two equations into (9), the relationship $\sigma(0) = \sigma(1) \approx 1/2$ is obtained. Taking into account that $r_2^+(1) \approx 0$ and $r_2^-(0) \approx 0$, the average current $I(V, V_G)$ through the two junctions is then evaluated as

$$
\begin{aligned}
I(0, -e/2C_G) &= -e \sum_{N=-\infty}^{\infty} \sigma(N)[r_2^+(N) - r_2^-(N)] \approx -e[\sigma(0) r_2^+(0) - \sigma(1) r_2^-(1)] \\
&\approx (e/2)[r_2^-(1) - r_2^+(0)]. \qquad (10)
\end{aligned}
$$

Since we already know the key relation for ATBs, i.e., $r_2^-(1) > r_2^+(0)$, we finally note that

$$
I(0, -e/2C_G) > 0. \qquad (11)
$$

This final expression indicates that a zero-bias current flows in a SET with identical ATBs at finite temperatures and for finite fixed gate voltages. Note that no electron heating occurs in the transport process described here, i.e. the temperature T is not increased, since electrons which have tunnelled across each junction biased well below the Coulomb gap do not release kinetic energy. In order to obtain a large current I in (10), $r_2^-(1)$ must be much larger than $r_2^+(0)$, and thus the effective thickness of the two barriers for a tunnelling electron at the Fermi level should be rather different for the two charge states $N = 0, 1$. In the case in which trapezoidal tunnelling barriers are used as the ATBs, considering the situation defined in (7), this requirement for $I(0, -e/2C_G) \gg 0$ is satisfied when

$$\phi_2 < eV_c < \phi_1. \tag{12}$$

Here we assume that the last terms of the image potential $\Delta\phi(z)$ in equation (7) are negligible.

Even if the bias voltage V is slightly decreased and becomes negative, a positive current still flows through the two junctions, namely

$$I(V, -e/2C_G) > 0 \qquad V < 0 \tag{13}$$

which, at low temperatures, becomes

$$\begin{aligned} I(V, -e/2C_G) &= -e \sum_{N=-\infty}^{\infty} \sigma(N)[r_2^+(N) - r_2^-(N)] \\ &\approx -e[\sigma(0)r_2^+(0) - \sigma(1)r_2^-(1)] > 0 \qquad V < 0. \end{aligned} \tag{14}$$

Substituting (9) into (14), we find that equation (13), describing 'electron pumping', requires that

$$\frac{r_1^-(1)}{r_1^+(0)} < \frac{r_2^-(1)}{r_2^+(0)}. \tag{15}$$

On decreasing the bias voltage V from zero, the ratio $r_1^-(1)/r_1^+(0)$ increases, while the ratio $r_2^-(1)/r_2^+(0)$ decreases. When the ratio $r_1^-(1)/r_1^+(0)$ just reaches the ratio $r_2^-(1)/r_2^+(0)$, the current through the two junctions stops flowing.

In figure 2 we show simulated transport properties of a SET with identical trapezoidal tunnelling barriers, choosing the following set of parameters: $C = C_G = 0.1$ aF, $d = 18$ nm, $\phi_1 = 8\phi_2 = 0.8$ eV, $m^* = 0.07$, and $\kappa = 10$. In this case one obtains $V_c = 0.27$ V satisfying the condition of large zero-bias current of (12). Figure 2(a) shows the I–V_G characteristics for $V = 0$. The current peaks are seen at $C_G V_G \sim m(e/2)$, as discussed above. Figure 2(b) shows the current versus bias voltage (I–V) characteristics for different fixed values of V_G. The zero-current offset is seen at $C_\Sigma V \sim -0.1$, i.e. $V \sim -0.01$ V.

In summary, we predict a zero-bias current to be observable in a SET with identical ATBs at finite temperatures and for finite fixed gate voltages. Also, it will be possible for the bias voltage V and the current I to have opposite signs, i.e. our particular SET will be able to act as an electron pump.

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